Tree-level unitarity constraints for the SM-like 2HDM

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Abstract

We consider a $\mathbb{Z}_2$-symmetric Two Higgs Doublet Model (2HDM), where the vacuum expectation values of both doublets are non-vanishing. Unitarity constraints for the SM-like 2HDM are analyzed for the mass of the lightest Higgs boson in the range [115, 127] GeV. New stringent bounds on $\tan\beta$, $0.17 \leqslant \tan\beta \leqslant 6.10$, are derived. A discussion of the source of these constraints is provided.

1 Introduction

Unitarity constraints for the Two Higgs Doublet Model (2HDM) have been extensively studied before [1, 2, 3, 4, 5, 6, 7, 8, 9, 10], however the SM-like limit of the model has been considered only in the case with violation of $\mathbb{Z}_2$ symmetry [11]. Bounds on the scalar particles' masses following from the basic assumption of tree-level unitarity in different versions of 2HDM, including the Inert Model, have been considered recently by one of us [12]. Here we present an extension of this analysis for the SM-like 2HDM. We assume that the mass of the lightest Higgs boson is in the range [115, 127] GeV in agreement with the newest results from the LHC for the SM Higgs boson presented in December 2011 [13].

2 The general and the SM-like Mixed Models

We consider a 2HDM with a general $\mathbb{Z}_2$-symmetric potential for the $SU(2)$ doublets ϕ_S, ϕ_D [14]:

$$\begin{aligned} V \;=\; & -\tfrac{1}{2}\Big[m_{11}^2(\phi_S^\dagger\phi_S) + m_{22}^2(\phi_D^\dagger\phi_D)\Big] + \tfrac{1}{2}\Big[\lambda_1(\phi_S^\dagger\phi_S)^2 + \lambda_2(\phi_D^\dagger\phi_D)^2\Big] + \\ & +\; \lambda_3(\phi_S^\dagger\phi_S)(\phi_D^\dagger\phi_D) + \lambda_4(\phi_S^\dagger\phi_D)(\phi_D^\dagger\phi_S) + \tfrac{1}{2}\lambda_5\Big[(\phi_S^\dagger\phi_D)^2 + (\phi_D^\dagger\phi_S)^2\Big]. \end{aligned} \tag{1}$$

The parameters m_{11}^2, m_{22}^2 and $\lambda_1 \dots \lambda_4$ are real numbers and without loss of generality [14, 15, 16] we can assume that λ_5 is real. For a stable vacuum state to exist it is necessary that [17]:

$$\text{a) } \lambda_1 > 0, \quad \lambda_2 > 0, \quad \text{b) } \lambda_3 + \sqrt{\lambda_1\lambda_2} > 0, \quad \text{c) } \lambda_{345} + \sqrt{\lambda_1\lambda_2} > 0, \tag{2}$$

where $\lambda_{345} = \lambda_3 + \lambda_4 + \lambda_5$.

We consider a model (called a Mixed Model) in which a Mixed vacuum is realized, i.e. in the state of the lowest energy (the global minimum of the potential) both of the doublets develop non-zero vacuum expectation values: $\langle\phi_S\rangle = v_S/\sqrt{2}$, $\langle\phi_D\rangle = v_D/\sqrt{2}$, $v_D/v_S = \tan\beta$, $\beta \in (0, \pi/2)$. The Mixed vacuum exists when the following conditions are satisfied [14, 12]

$$v_S^2 = \frac{m_{11}^2\lambda_2 - \lambda_{345}m_{22}^2}{\lambda_1\lambda_2 - \lambda_{345}^2} > 0, \quad v_D^2 = \frac{m_{22}^2\lambda_1 - \lambda_{345}m_{11}^2}{\lambda_1\lambda_2 - \lambda_{345}^2} > 0, \quad \lambda_4 + \lambda_5 < 0, \quad \lambda_5 < 0, \quad \lambda_1\lambda_2 - \lambda_{345}^2 > 0. \tag{3}$$

The matrix of second derivatives of the potential is non-diagonal, so the mass eigenstates are mixtures of the fields appearing in the potential. There is one mixing angle β in the charged and the CP-odd sectors and one mixing angle α in the CP-even sector, $\alpha \in (-\pi/2, \pi/2)$. There arise 3 Goldstone bosons and 5 physical Higgs particles: $H^\pm$, A, H, h, with $M_H > M_h$. Different models of Yukawa interactions can be chosen, however we do not fix a particular one here, because Yukawa interactions do not effect the following analysis.

When an additional condition:

$$\sin(\beta - \alpha) = 1 \tag{4}$$

is imposed, h couples to gauge bosons at the tree-level like the SM Higgs particle[1] [18]. Then, it is justified to apply the following experimental bounds on its mass,

$$M_h \in [115,\ 127]\,\text{GeV}, \tag{5}$$

found for the SM Higgs boson [13]. A Mixed Model with additional assumptions (4) and (5) we call a SM-like Mixed Model. In the following we will focus on this model, however, for comparison results for the general Mixed Model are presented as well.

[1] And so it does to fermions, if e.g. Model II of Yukawa interactions is chosen.

3 Tree-level unitarity approach

To derive constraints from unitarity we follow the standard high-energy approach, where longitudinally polarized states of vector bosons are replaced by the corresponding would-be Goldstone bosons [19] and only the quartic interactions are included in the $2 \to 2$ scatterings. In this analysis we consider the scattering matrix for the original fields of the Lagrangian, i.e. not mass eigenstates, as this makes analysis simpler [6].

Inequalities resulting from the unitarity condition on the s-wave

$$|\Re(a^{(0)})| < \frac{1}{2} \tag{6}$$

are considered. They are inferred from the full tree-level high-energy scattering matrix of the scalar sector with 25 different channels [12]. The inequalities are solved numerically (as in [7]) taking into account explicitly the positivity constraints (2) and conditions necessary for the existence of Mixed vacuum (3)[2], as well as two additional constraints (4) and (5) for the SM-like Mixed Model. For simplicity the set of the relevant conditions, namely: tree-level unitarity constraint (6) together with constraints (2) and (3) (for the SM-like Model the set contains in addition two constraints (4) and (5)) will be called the unitarity constraints.

A scan, subject to the unitarity constraints, over different values of $\tan\beta$ and Higgs boson masses is performed in the following ranges:

$$\tan\beta \in [0,\, 60]^3,\;\; M_A,\, M_H,\, M_{H^\pm} \in [0,\, 720]\,\mathrm{GeV},\; M_h \in [0,\, M_H]. \tag{7}$$

4 Results

We perform a scan for the general and the SM-like Mixed Models and as a result bounds on scalars' masses as well as on $\tan\beta$ are obtained.

4.1 General Mixed Model

Unitarity constraints lead to the following upper bounds on the Higgs bosons' masses in the general Mixed Model:

$$\begin{aligned} M_{H^\pm} &\leqslant 690\,\mathrm{GeV},\\ M_A &\leqslant 711\,\mathrm{GeV},\\ M_H &\leqslant 688\,\mathrm{GeV},\\ M_h &\leqslant 499\,\mathrm{GeV}. \end{aligned} \tag{8}$$

Note that h is remarkably lighter than the other Higgs bosons and that $\tan\beta$ is not bounded by the unitarity constraints. These results agree at the level of 1-3% with the most precise analytical results obtained in Ref. [10].

4.2 SM-like Mixed Model

For the SM-like Mixed Model we perform an analysis analogous to the one for the general case discussed above. The following upper bounds are found:

$$\begin{aligned} M_{H^\pm} &\leqslant 616\,\mathrm{GeV},\\ M_A &\leqslant 711\,\mathrm{GeV},\\ M_H &\leqslant 609\,\mathrm{GeV}, \end{aligned} \tag{9}$$

and

$$0.17 \leqslant \tan\beta \leqslant 6.10. \tag{10}$$

We see that the bounds for $H^\pm$ and H are lowered by 70-80 GeV in comparison to the general case (compare with Eq. (8)). What definitely draws attention is the stringent bound on $\tan\beta$ which arises in this case. In the next section we discuss main sources of this strong bound. The regions of masses allowed by the unitarity constraints and their correlations with $\tan\beta$ are presented in Fig. 1 and Fig. 2 (pale area), respectively. The upper bounds, Eqs. (8) and (9), should be treated as maximal values reached during the scan. It should be underlined that these values not always can be approached simultaneously, as it can be inferred from the Figs. 1 and 2, where allowed regions in respective parameter spaces are presented.

The bounds presented above suffer from some uncertainties inherent to the numerical method in use. The uncertainties can be evaluated using Figs. 1 and 2. If the boundaries of the regions of allowed masses are smooth and well filled with points the uncertainties are small (should not exceed 1%), while if the boundaries consist of separated points, the uncertainties are obviously larger[4].

[2] It is not obvious if they were taken into account in some earlier analyses.

[3] Although, there exists a lower bound on $\tan\beta$ which comes from the assumption of perturbativity of the $\bar{t}bH^\pm$ coupling and is valid in the models I-IV of Yukawa interactions [20], we do not impose it not to obscure the effects of the considered constraints.

[4] They can be estimated as being of order of the distances between nearest points divided by the upper bound on respective mass.

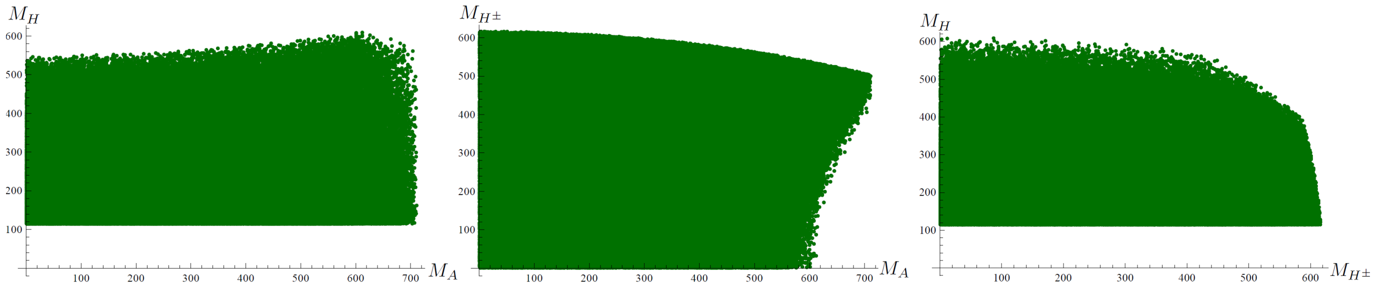

Figure 1: Regions of masses allowed in the SM-like Mixed Model by the unitarity constraints. M_h does not display significant correlations with masses of other Higgses, thus corresponding plots are not shown.

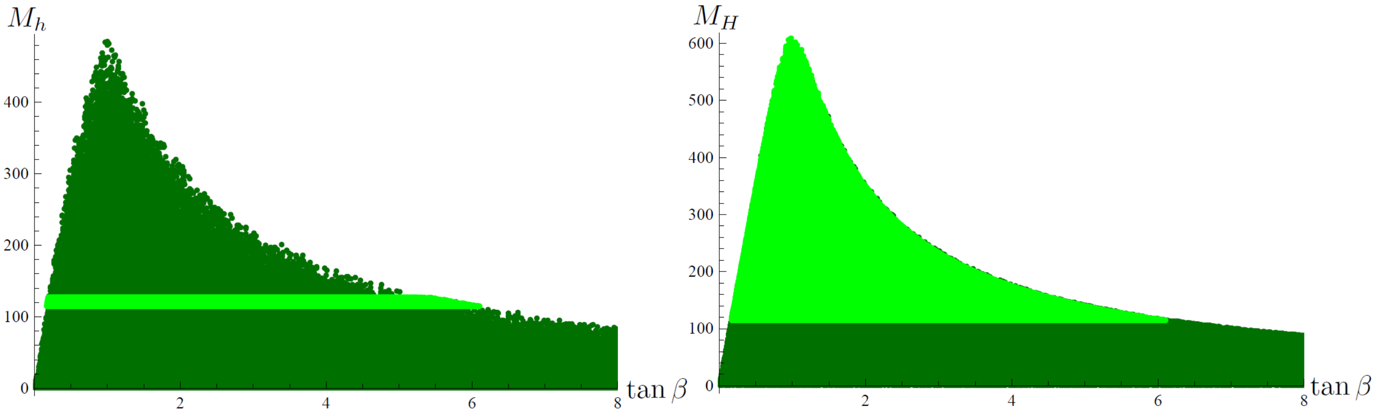

Figure 2: Regions of masses: M_h (left) and M_H (right) versus $\tan\beta$ allowed in the SM-like Mixed Model by the unitarity constraints (pale points). For comparison results of a scan without experimental bounds imposed on M_h are presented (dark points). The remaining regions of allowed masses do not exhibit dependence on $\tan\beta$.

5 Discussion of the bound on $\tan\beta$ in the SM-like Mixed Model

5.1 The role of the experimental bound on M_h

The correlations between the masses and $\tan\beta$ allowed by the unitarity constraints in the SM-like Mixed Model are presented in Fig. 2 (pale area). For comparison analogous results obtained without condition (5) imposed are presented (Fig. 2, dark area)[5].

The mass of h can be expressed in the Mixed Models as [14]:

$$M_h^2 = \frac{v^2}{2}\frac{1}{1+\tan^2\beta}\left(\lambda_1 + \lambda_2\tan^2\beta - \sqrt{(\lambda_1-\lambda_2\tan^2\beta)^2 + 4\lambda_{345}^2\tan^2\beta}\right).$$

Hence for fixed λ's we have

$$M_h^2 \to \frac{v^2}{2}\left(\lambda_2 - \sqrt{\lambda_2^2}\right) = 0 \text{ for } \tan\beta\to\infty \quad \text{and} \quad M_h^2 \to \frac{v^2}{2}\left(\lambda_1 - \sqrt{\lambda_1^2}\right) = 0 \text{ for } \tan\beta \to 0. \tag{11}$$

Therefore $\tan\beta$ cannot be neither too small nor too large if the lower limit on M_h is applied. In general, an upper bound on $\tan\beta$ will arise whenever a lower bound on M_h is imposed.

In Fig. 2 (left panel, pale area) the allowed region for M_h and $\tan\beta$ is shown. It is clear that the upper and lower limits on $\tan\beta$ are correlated with lower limit $M_h = 114\,\text{GeV}$, the same holds for M_H (Fig. 2, right panel, pale area).

These results are in agreement with the reasoning (11) and confirm that the bounds on $\tan\beta$, Eq. (10), are due to the experimental lower bound on M_h mass (5).

5.2 The role of the condition $\sin(\beta-\alpha) = 1$

If we waive the constraint (4) $\sin(\beta-\alpha)=1$ but keep the experimental limits (5) on M_h, the bounds on $\tan\beta$ are found to be almost the same as in Eq. (10), namely $0.17 \leqslant \tan\beta \leqslant 6.11$. Thus, the bounds on $\tan\beta$ are arise mainly due to the constrain on M_h and it is not necessary that h couples to gauge bosons and fermions like the SM Higgs to obtain them.

[5]The large $\tan\beta$ regions have not been included in the plot, because the maximal allowed Higgs boson masses monotonically fall down with raising $\tan\beta$ to approach the values $M_h \approx M_H \approx 12\,\text{GeV}$.

6 Discussion and summary

We considered the tree-level unitarity for the SM-like Mixed Model, and found very stringent bounds on $\tan\beta$: $0.17 \leqslant \tan\beta \leqslant 6.10$. These bounds have been shown to be induced by the lower experimental limit (115 GeV) imposed on M_h. It is worth noticing that the limits on $\tan\beta$ are obtained without specifying the type of the Yukawa interactions and without applying constraints *directly* on couplings with gauge bosons (i.e. on $\sin(\beta-\alpha)$).

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